Revision Sheet: Confidence Intervals and Sampling Distribution

1. Purpose of Confidence Intervals

- Statistical Inference: Estimate the population mean (μ) based on a sample.

- Confidence Interval (CI): Provides a range that likely contains μ , given a confidence level (e.g., 95%).

2. Sampling Distribution of the Sample Mean

- Given:

- Population mean: µ
- Population standard deviation: $\boldsymbol{\sigma}$
- Sample size: n
- Properties:
 - Mean of X: $E(X) = \mu$
 - Standard Deviation of X: SD(X) = σ / \sqrt{n}

- Central Limit Theorem (CLT): As n increases, the sampling distribution of X approaches a normal distribution, N(μ , σ^2/n), regardless of the population distribution (usually n \geq 30 is sufficient).

3. Standardizing the Sample Mean

- Z-score for X:

 $Z = (X - \mu) / (\sigma / \sqrt{n})$

- This allows us to use the standard normal distribution to calculate probabilities and construct confidence intervals.

4. Constructing a Confidence Interval for **µ**

- Formula for a CI when σ is known:

$\mu \in (X - Z\alpha/2 * (\sigma / \sqrt{n}), X + Z\alpha/2 * (\sigma / \sqrt{n}))$

- For a 95% confidence level: Za/2 = 1.96.

- For different confidence levels, adjust Za/2 accordingly.

5. Interpreting the Confidence Interval

- A 95% CI means if we repeatedly sampled, 95% of the intervals would contain $\mu.$

- This interval does ***not*** imply a 95% probability that the specific interval contains μ .

Population Mean Problems

Problem 1: Suppose we sample 30 data points from a normal distribution with a mean of 70 and a standard deviation of 3.

• a) What is the probability that the sample mean is greater than 72?

• b) Find the value of kk such that P(k<X)=0.15.

Problem 2: Suppose we take a sample of 40 data points from a normal distribution with a mean of 150 and a standard deviation of 4.

• a) What is the probability that the sample mean is less than 148?

• b) Find kk such that P(k>X)=0.1

Confidence Intervals

Problem 1: A study measured the concentration of a chemical in 20 water samples and found an average concentration of 5.8 mg/L. The population standard deviation is assumed to be 0.4 mg/L.

- a) Determine a 90% confidence interval for the population mean.
- b) Determine a 95% confidence interval for the population mean.
- c) Determine a 99% confidence interval for the population mean.

Problem 2: The average time taken to complete a task by a sample of 16 employees was found to be 45.6 minutes. The population standard deviation is assumed to be 2.2 minutes.

• a) Determine a 90% confidence interval for the population mean.

• b) Determine a 95% confidence interval for the population mean.

Additional Practice Problems

Problem 1: A sample of 12 trees in a forest was measured and found to have an average height of 30.2 feet. Assume the population standard deviation is 3 feet.

• a) Determine a 95% confidence interval for the average height of trees in the forest.

• b) Determine a 98% confidence interval for the average height of trees in the forest.

Problem 2: A batch of 25 bulbs was tested for lifespan, yielding an average lifespan of 1200 hours. Assume the population standard deviation is 50 hours.

• a) Determine a 90% confidence interval for the population mean lifespan of the bulbs.

• b) Determine a 95% confidence interval for the population mean lifespan of the bulbs.